DYNAMIC LEARNING MAPS ESSENTIAL ELEMENTS

FOR

Mathematics

Background on the Dynamic Learning Maps Essential Elements

The Dynamic Learning Maps Essential Elements are specific statements of knowledge and skills linked to the grade-level expectations identified in the Common Core State Standards. The purpose of the Dynamic Learning Maps Essential Elements is to build a bridge from the content in the Common Core State Standards to academic expectations for students with the most significant cognitive disabilities. The initial draft of the Dynamic Learning Maps Essential Elements (then called the Common Core Essential Elements) was released in the spring of 2012.

The initial version of the Dynamic Learning Maps Essential Elements (DLM EEs) was developed by a group of educators and content specialists from the 12 member states of the Dynamic Learning Maps Alternate Assessment Consortium (DLM) in the spring of 2011. Led by Edvantia, Inc., a sub-contractor of DLM, representatives from each state education agency and the educators and content specialists they selected developed the original draft of DLM EEs. Experts in mathematics and English language arts, as well as individuals with expertise in instruction for students with significant cognitive disabilities reviewed the draft documents. Edvantia then compiled the information into the version released in the spring of 2012.

Concurrent with the development of the DLM EEs, the DLM consortium was actively engaged in building learning maps in mathematics and English language arts. The DLM learning maps are highly connected representations of how academic skills are acquired, as reflected in research literature. In the case of the DLM project, the Common Core State Standards helped to specify academic targets, while the surrounding map content clarified how students could reach the specified standard. Learning maps of this size had not been previously developed, and as a result, alignment between the DLM EEs and the learning maps was not possible until the fall of 2012, when an initial draft of the learning maps was available for review.

Alignment of the DLM EEs to the DLM Learning Maps

Teams of content experts worked together to revise the initial version of the DLM EEs and the learning maps to ensure appropriate alignment of these two critical elements of the project. Alignment involved horizontal alignment of the DLM EEs with the Common Core State Standards and vertical alignment of the DLM EEs with meaningful progressions in the learning maps. The alignment process began when researchers Caroline Mark and Kelli Thomas compared the learning maps with the initial version of the DLM EEs to determine how the map and the DLM EEs should be adjusted to improve their alignment. The teams of content experts most closely involved with this alignment work included:

Mathematics	English Language Arts
Kelli Thomas, Ph.D. (co-lead)	Caroline Mark, Ph.D. (lead)
Angela Broaddus, Ph.D. (co-lead)	Jonathan Schuster, Ph.D.
Perneet Sood	Russell Swinburne Romine, Ph.D.
Kristin Joannou	Suzanne Peterson
Bryan Candea Kromm	

These teams worked in consultation with Sue Bechard, Ph.D. and Karen Erickson, Ph.D., who offered guidance based on their experience in alternate assessments of students with significant cognitive disabilities.

The Alignment Process

The process of aligning the learning map and the DLM EEs began by identifying nodes in the maps that represented the essential elements in mathematics and English language arts. This process revealed areas in the maps where additional nodes were needed to account for incremental growth reflected from an essential element in one grade to the next. Also identified were areas in which an essential element was out of place developmentally, according to research, with other essential elements. For example, adjustments were made when an essential element related to a higher-grade map node appeared earlier on the map than an essential element related to a map node from a lower grade (e.g., a fifth-grade skill preceded a third-grade skill). Finally, the alignment process revealed DLM EEs that were actually written as instructional tasks rather than learning outcomes.

This initial review step provided the roadmap for subsequent revision of both the learning maps and the DLM EEs. The next step in the DLM project was to develop the claims document, which served as the basis for the evidence-centered design of the DLM project and helped to further refine both the modeling of academic learning in the maps and the final revisions to the DLM EEs.

Claims and Conceptual Areas

The DLM system uses a variant of evidence-centered design (ECD) as the framework for developing the DLM Alternate Assessment System. While ECD is multifaceted, it starts with a set of claims regarding important knowledge in the domains of interest (mathematics and English language arts), as well as an understanding of how that knowledge is acquired. Two sets of claims have been developed for DLM that identify the major domains of interest within mathematics and English language arts for students with significant cognitive disabilities. These claims are broad statements about expected student learning that serve to focus the scope of the assessment. Because the learning map identifies particular paths to the acquisition of academic skills, the claims also help to organize the structures in the learning map for this population of students. Specifically, conceptual areas within the map further define the knowledge and skills required to meet the broad claims identified by DLM.

The claims are also significant because they provide another means through which to evaluate alignment between the DLM EEs and the learning map nodes, and serve as the foundation for evaluating the validity of inferences made from test scores. DLM EEs related to a particular claim and conceptual area must clearly link to one another, and the learning map must reflect how that knowledge is acquired. Developing the claims and conceptual areas for DLM provided a critical framework for organizing nodes on the learning maps and, accordingly, the DLM EEs that align with each node.

The table below reveals the relationships among the claims, conceptual areas, and DLM EEs in mathematics. The DLM EEs are represented with codes that reflect the domains in mathematics. For example, the first letter or digit represents the grade of record, the next code reflects the domain, followed by the number that aligns with the Common Core State Standard grade level expectation. As such, K.CC.1 is the code for the DLM EE that aligns with kindergarten (K), counting and cardinality (CC), standard 1. Keys to the codes can be found under the table.

Clearly articulated claims and conceptual areas for DLM served as an important evidence-centered framework within which this version of the DLM EEs was developed. With the claims and conceptual areas in place, the relationship between DLM EEs within a claim and conceptual area or across grade levels is easier to track and strengthen. The learning maps, as well as the claims and conceptual areas, had not yet been developed when the original versions of the DLM EEs were created. As such, the relationship of DLM EEs within and across grade levels was more difficult to evaluate at that time.

Table 1. Dynamic Learning Maps Claims and Conceptual Areas for Students with Significant Cognitive Disabilities in Mathematics

Claim 1	**Number Sense: Students demonstrate increasingly complex understanding of number sense.** Conceptual Areas in the Dynamic Learning Map: **MC 1.1 Understand number structures (counting, place value, fraction)** *Essential Elements Included: K.CC.1, 4 ,5; 1.NBT.1a-b; 2.NBT.2a-b,3; 3.NBT.1,2,3; 3.NF.1-3; 4.NF.1-2,3; 5.NF.1,2; 6.RP.1; 7.RP.1-3; 7.NS.2.c-d; 8.NS.2.a* **MC 1.2 Compare, compose, and decompose numbers and sets** *Essential Elements Included: K.CC.6; 1.NBT.2, 3, 4, 6; 2.NBT.1, 4, 5b; 4.NBT.2, 3; 5.NBT.1, 2, 3, 4; 6.NS.1, 5-8; 7.NS.3; 8.NS.2.b; 8.EE.3-4;* **MC 1.3 Calculate accurately and efficiently using simple arithmetic operations** *Essential Elements Included: 2.NBT.5.a, 6-7; 3.OA.4; 4.NBT.4, 5.NBT.5, 6-7; 6.NS.2, 3; 7.NS.1, 2.a, 2.b; 8.NS.1; 8.EE.1; N-CN.2.a, 2.b, 2.c; N-RN.1; S-CP.1-5; S-IC.1-2*
Claim 2	**Geometry: Students demonstrate increasingly complex spatial reasoning and understanding of geometric principles.** Conceptual Areas in the Dynamic Learning Map: **MC 2.1 Understand and use geometric properties of two- and three-dimensional shapes** *Essential Elements Included: K.MD.1-3; K.G.2-3; 1.G.1, 2; 2.G.1; 3.G.1; 4.G.1, 2; 4.MD.5, 6; 5.G.1-4; 5.MD.3; 7.G.1, 2, 3, 5; 8.G.1, 2, 4, 5; G-CO.1, 4-5, 6-8; G-GMD.1-3, 4* **MC 2.2 Solve problems involving area, perimeter, and volume** *Essential Elements Included: 1.G.3; 3.G.2; 4.G.3; 4.MD.3; 5.MD.4-5; 6.G.1, 2; 7.G.4, 6; 8.G.9; G-GMD.1-3; G-GPE.7*
Claim 3	**Measurement Data and Analysis: Students demonstrate increasingly complex understanding of measurement, data, and analytic procedures.** Conceptual Areas in the Dynamic Learning Map: **MC 3.1 Understand and use measurement principles and units of measure** *Essential Elements Included: 1.MD.1-2, 3.a, 3.b, 3.c, 3.d; 2.MD.1, 3-4, 5, 6, 7, 8; 3.MD.1, 2, 4; 4.MD.1, 2.a, 2.b, 2.c, 2.d; 5.MD.1.a, 1.b, 1.c; N-Q.1-3* **MC 3.2 Represent and interpret data displays** *Essential Elements Included: 1.MD.4; 2.MD.9-10; 3.MD.3; 4.MD.4.a, 4.b; 5.MD.2; 6.SP.1-2, 5; 7.SP.1-2, 3, 5-7; 8.SP.4; S-ID. 1-2, 3, 4*
Claim 4	**Algebraic and functional reasoning: Students solve increasingly complex mathematical problems, making productive use of algebra and functions.** Conceptual Areas in the Dynamic Learning Map: **MC 4.1. Use operations and models to solve problems** *Essential Elements Included: K.OA.1, 1.a, 1.b, 2, 5.a, 5.b; 2.OA.3, 4; 3.OA.1-2, 8; 4.OA.1-*

	2, 3, 4; 6.EE.1-2, 3, 5-7; 7.EE.1, 4; 8.EE.7; A-CED.1, 2-4; A-SSE.1, 3 **MC 4.2 Understand patterns and functional thinking** *Essential Elements Included: 3.OA.9; 4.OA.5; 5.OA.3; 7.EE.2; 8.EE.5-6; 8.F.1-3, 4, 5; A-REI.10-12; A-SSE.4; F-BF.1, 2; F-IF.1-3, 4-6; F-LE.1*

A-CED = creating equations; A-SSE = seeing structure in equations BF = building functions; CC = counting & cardinality; EE = expressions & equations; F-BF = basic fractions; F-IF = interpreting functions; G = geometry; G-GMD = geometric measurement & dimension; G-GPE = general properties & equations; MD = measurement & data; NBT = numbers & operations in base ten; N-CN = complex number system; NF = numbers & operations - fractions; N-RN = real number system; NS = number systems; N-Q = number & quantity; OA = operations & algebraic thinking; RP = ratios & proportional relationships; S-IC- statistics & probability - making inferences/justifying conclusions; S-ID = statistics & probability - interpreting categorical & quantitative data; SP = statistics & probability

Resulting Changes to the DLM Essential Elements

The development of the entire DLM Alternate Assessment System guided a final round of revisions to the DLM EEs, which can be organized into four broad categories: alignment across grade levels, language specificity, common core alignment, and defining learning expectations (rather than instructional tasks). The first type of revision was required to align the DLM EEs across grade levels, both vertically and horizontally. The maps, and the research supporting them, were critical in determining the appropriate progression of skills and understandings from grade to grade. This alignment across grade levels was important within and across standards, strands, and domains. For example, in determining when it was appropriate to introduce concepts in mathematics regarding the relative position of objects, we had to consider the grade level at which prepositions that describe relative position were introduced in English language arts. Examining the research-based skill development outlined in the learning map aided in these kinds of determinations.

The articulation of the claims and conceptual areas reinforced the need for specific language in the DLM EEs to describe learning within an area. Because teams assigned to grade bands developed the first round of DLM EEs, the language choices from one grade to the next were not consistent. Even when closely related skills, concepts, or understandings were targeted, the same terms were not always selected to describe the intended learning outcome. The teams of content experts who worked on this revised version of the DLM EEs were very intentional in selecting a common set of terms to reflect the claims and conceptual areas and applied them consistently across the entire set of DLM EEs.

Another important change in this version of the DLM EEs involved alignment to the Common Core State Standards (CCSS). Given that the DLM EEs are intended to clarify the bridge to the CCSS expectations for students with the most significant cognitive disabilities, it is critical that alignment be as close as possible without compromising learning and development over time. While there was never a one-to-one correspondence between the CCSS and the DLM EEs, the revisions have made the alignment between the two more precise than it was in the first version.

Finally, revisions to the DLM EEs involved shifting the focus of a small number of DLM EEs that were written in the form of instructional tasks rather than learning expectations, and adding “With guidance and support” to the beginning of a few of the DLM EEs in the primary grades in English language arts to reflect the expectations articulated in the CCSS.

Members of the DLM consortium reviewed each of the changes to the original version of the DLM EEs. Four states provided substantive feedback on the revisions, and this document incorporates the changes those teams suggested.

Access to Instruction and Assessment

The DLM EEs specify learning targets for students with significant cognitive disabilities; however, they do not describe all of the ways that students can engage in instruction or demonstrate understanding through an assessment. Appropriate modes of communication, both for presentation or response, are not stated in the DLM EEs unless a specific mode is an expectation. Where no limitation has been stated, no limitation should be inferred. Students' opportunities to learn and to demonstrate learning during assessment should be maximized by providing whatever communication, assistive technologies, augmentative and alternative communication (AAC) devices, or other access tools that are necessary and routinely used by the student during instruction.

Students with significant cognitive disabilities include a broad range of students with diverse disabilities and communication needs. For some students with significant cognitive disabilities, a range of assistive technologies is required to access content and demonstrate achievement. For other students, AAC devices or accommodations for hearing and visual impairments will be needed. During instruction, teams should meet individual student needs using whatever technologies and accommodations are required. Examples of some of the ways that students may use technology while learning and demonstrating learning are topics for professional development, and include:

- communication devices that compensate for a student's physical inability to produce independent speech.
- alternate access devices that compensate for a student's physical inability to point to responses, turn pages in a book, or use a pencil or keyboard to answer questions or produce writing.

Guidance and Support

The authors of the CCSS use the words "prompting and support" at the earliest grade levels to indicate when students are not expected to achieve standards completely independently. Generally, "prompting" refers to "the action of saying something to persuade, encourage, or remind someone to do or say something" (McKean, 2005). However, in special education, prompting is often used to mean a system of structured cues to elicit desired behaviors that otherwise would not occur. In order to clearly communicate that teacher assistance is permitted during instruction of the DLM EEs and is not limited to structured prompting procedures, the decision was made by the stakeholder group to use the more general term *guidance* throughout the DLM EEs.

Guidance and support during instruction should be interpreted as teacher encouragement, general assistance, and informative feedback to support the student in learning. Some examples of the kinds of teacher behaviors that would be considered guidance and support include verbal supports, such as

- getting the student started (e.g., "Tell me what to do first."),
- providing a hint in the right direction without revealing the answer (e.g., Student wants to write *dog* but is unsure how, so the teacher might say, "See if you can write the first letter in the word, /d/og [phonetically pronounced]."),
- using structured technologies such as task-specific word banks, or
- providing structured cues such as those found in prompting procedures (e.g., least-to-most prompts, simultaneous prompting, and graduated guidance).

Guidance and support as described above applies to instruction and is also linked to demonstrating learning relative to DLM EEs, where guidance and support is specifically called out within the standards.

Conclusion

Developing the research-based model of knowledge and skill development represented in the DLM Learning Maps supported the articulation of assessment claims for mathematics and English language arts. This articulation subsequently allowed for a careful revision of the DLM EEs to reflect both horizontal alignment with the CCSS and vertical alignment across the grades, with the goal of moving students toward more sophisticated understandings in both domains. Though the contributions made by Edvantia and our state partners in developing the initial set of DLM EEs were a critical first step, additional revisions to the DLM EEs were required to ensure consistency across all elements of the Dynamic Learning Maps Alternate Assessment System.

APPENDIX

Development of the Dynamic Learning Maps Essential Elements has been a collaborative effort among practitioners, researchers, and our state representatives. Listed below are the reviews and the individuals involved with each round of improvements to the Dynamic Learning Maps Essential Elements. Thank you to all of our contributors.

Review of Draft Two of Dynamic Learning Maps Essential Elements

A special thanks to all of the experts nominated by their state to review draft two of the Dynamic Learning Maps Essential Elements. We are grateful for your time and efforts to improve these standards for students with significant cognitive disabilities. Your comments have been incorporated into this draft. The states with teams who reviewed draft two include:

Illinois	Oklahoma
Iowa	Utah
Kansas	Virginia
Michigan	West Virginia
Missouri	Wisconsin

Development of the Original Dynamic Learning Maps Common Core Essential Elements

A special thanks to Edvantia and the team of representatives from Dynamic Learning Maps consortium states who developed the original Common Core Essential Elements upon which the revised Dynamic Learning Maps Essential Elements are based. The team from Edvantia who led the original effort included:

Jan Sheinker, Sheinker Educational Services, Inc.
Beth Judy, Director, Assessment, Alignment, and Accountability Services
Nathan Davis, Information Technology Specialist
Kristen Deitrick, Corporate Communications Specialist

Linda Jones, Executive Assistant

Representatives from Dynamic Learning Maps consortium states included:

IOWA
SEA Representatives: Tom Deeter, Emily Thatcher
Stakeholders: Peggy Akins, Judy Hamer, Kathleen Kvamme-Promes, Donna Shaw

KANSAS
SEA Representatives: Debbie Matthews, Kris Shaw
Stakeholders: Debby Byrne, Holly Draper, Dawn Gresham, Linda Hickey

MICHIGAN
SEA Representatives: Joanne Wilkelman, Adam Wyse
Stakeholders: Debra Susan Asano, Thomai Gersh, Marcia O'Brian, Terri Portice

MISSOURI
SEA Representatives: Lynn Everett, Jane VanDeZande
Stakeholders: Melia Franklin, Lou Ann Hoover, Debbie Jameson, Kate Sadler

NEW JERSEY
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Stakeholders: Brenda Berrios, Neal Webster, Tina Yurcho

NORTH CAROLINA
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Stakeholders: Emma Hatfield-Sidden, Judy Jennings, Jennifer Michalenok,

OKLAHOMA
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Stakeholders: Pam Cox, Dianna Daubenspeck, Sondra LeGrande, Christie Stephenson

UTAH
SEA Representatives: Wendy Carver, Kurt Farnsworth
Stakeholders: James Bray, Janice Hill, Linda Stallviere, Ryan Webb

VIRGINIA
SEA Representative: John Eisenberg
Stakeholders: Maria Beck, Daniel Blegun, Al Klugh, Cheryl Ann Prevatte

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SEA Representatives: Judy Kraft, Janice Tornow
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Stakeholders: Robert Bartlett, Gerald Hartley, Angel Harris, Angela See

WISCONSIN
SEA Representatives: Emilie Amundson, Kristen Burton
Stakeholders: Lori Hillyer, Tamara Maxwell, Connie Persike, Sara Vold

High School Mathematics Domain: Statistics and Probability★—Interpreting Categorical and Quantitative Data

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Summarize, represent, and interpret data on a single count or measurement variable.	
S-ID.1. Represent data with plots on the real number line (dot plots, histograms, and box plots).	**EE.S-ID.1–2.** Given data, construct a simple graph (line, pie, bar, or picture) or table, and interpret the data.
S-ID.2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	
S-ID.3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	**EE.S-ID.3.** Interpret general trends on a graph or chart.
S-ID.4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	**EE.S-ID.4.** Calculate the mean of a given data set (limit the number of data points to fewer than five).
CLUSTER: Summarize, represent, and interpret data on two categorical and quantitative variables.	
S-ID.5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	Not applicable. See **EE.F-IF.1** and **EE.A-REI.6–7**.
S-ID.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.	Not applicable.
S-ID.6.a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions, or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*	
S-ID.6.b. Informally assess the fit of a function by plotting and analyzing residuals.	
S-ID.6.c. Fit a linear function for a scatter plot that suggests a linear association.	
CLUSTER: Interpret linear models.	
S-ID.7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	Not applicable. See **EE.F-IF.4–6**.
S-ID.8. Compute (using technology), and interpret the correlation coefficient of a linear fit.	Not applicable.

CCSS Grade-Level Standards	DLM Essential Elements
S-ID.9. Distinguish between correlation and causation.	Not applicable.

High School Mathematics Domain: Statistics and Probability—Making Inferences and Justifying Conclusions

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand and evaluate random processes underlying statistical experiments.	
S-IC.1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	**EE.S-IC.1–2.** Determine the likelihood of an event occurring when the outcomes are equally likely to occur.
S-IC.2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*	
CLUSTER: Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	
S-IC.3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	Not applicable. See **EE.S-ID.1–2**.
S-IC.4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	Not applicable. See **EE.S-ID.1–2**.
S-IC.5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	Not applicable. See **EE.S-ID.1–2**.
S-IC.6. Evaluate reports based on data.	Not applicable. See **EE.S-ID.1–2**.

High School Mathematics Domain: Statistics and Probability—Conditional Probability and the Rules of Probability

<table>
<tr><th>CCSS Grade-Level Standards</th><th>DLM Essential Elements</th></tr>
<tr><td colspan="2">CLUSTER: Understand independence and conditional probability, and use them to interpret data.</td></tr>
<tr><td>S-CP.1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).</td><td rowspan="5">EE.S-CP.1–5. Identify when events are independent or dependent.</td></tr>
<tr><td>S-CP.2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.</td></tr>
<tr><td>S-CP.3. Understand the conditional probability of A given B as $P(A \text{ and } B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.</td></tr>
<tr><td>S-CP.4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.</td></tr>
<tr><td>S-CP.5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.</td></tr>
<tr><td colspan="2">CLUSTER: Use the rules of probability to compute probabilities of compound events in a uniform probability model.</td></tr>
<tr><td>S-CP.6. Find the conditional probability of A given B as the fraction of B’s outcomes that also belong to A, and interpret the answer in terms of the model.</td><td>Not applicable.
See EE.S-IC.1–2.</td></tr>
<tr><td>S-CP.7. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) – P(A \text{ and } B)$, and interpret the answer in terms of the model.</td><td>Not applicable.
See EE.S-IC.1–2.</td></tr>
<tr><td>S-CP.8. (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.</td><td>Not applicable.</td></tr>
</table>

CCSS Grade-Level Standards	DLM Essential Elements
S-CP.9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.	Not applicable.

High School Mathematics Domain: Statistics and Probability★—Interpreting Categorical and Quantitative Data

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Summarize, represent, and interpret data on a single count or measurement variable.	
S-ID.1. Represent data with plots on the real number line (dot plots, histograms, and box plots).	**EE.S-ID.1–2.** Given data, construct a simple graph (line, pie, bar, or picture) or table, and interpret the data.
S-ID.2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	
S-ID.3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	**EE.S-ID.3.** Interpret general trends on a graph or chart.
S-ID.4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	**EE.S-ID.4.** Calculate the mean of a given data set (limit the number of data points to fewer than five).
CLUSTER: Summarize, represent, and interpret data on two categorical and quantitative variables.	
S-ID.5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	Not applicable. See **EE.F-IF.1** and **EE.A-REI.6–7**.
S-ID.6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.	Not applicable.
S-ID.6.a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions, or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*	
S-ID.6.b. Informally assess the fit of a function by plotting and analyzing residuals.	
S-ID.6.c. Fit a linear function for a scatter plot that suggests a linear association.	
CLUSTER: Interpret linear models.	
S-ID.7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	Not applicable. See **EE.F-IF.4–6**.
S-ID.8. Compute (using technology), and interpret the correlation coefficient of a linear fit.	Not applicable.

CCSS Grade-Level Standards	DLM Essential Elements
S-ID.9. Distinguish between correlation and causation.	Not applicable.

High School Mathematics Domain: Statistics and Probability—Making Inferences and Justifying Conclusions

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand and evaluate random processes underlying statistical experiments.	
S-IC.1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	**EE.S-IC.1–2.** Determine the likelihood of an event occurring when the outcomes are equally likely to occur.
S-IC.2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*	
CLUSTER: Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	
S-IC.3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	Not applicable. See **EE.S-ID.1–2**.
S-IC.4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	Not applicable. See **EE.S-ID.1–2**.
S-IC.5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	Not applicable. See **EE.S-ID.1–2**.
S-IC.6. Evaluate reports based on data.	Not applicable. See **EE.S-ID.1–2**.

High School Mathematics Domain: Statistics and Probability—Conditional Probability and the Rules of Probability

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand independence and conditional probability, and use them to interpret data.	
S-CP.1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	**EE.S-CP.1–5.** Identify when events are independent or dependent.
S-CP.2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	
S-CP.3. Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.	
S-CP.4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*	
S-CP.5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*	
CLUSTER: Use the rules of probability to compute probabilities of compound events in a uniform probability model.	
S-CP.6. Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.	Not applicable. See **EE.S-IC.1–2**.
S-CP.7. Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) – P(A \text{ and } B)$, and interpret the answer in terms of the model.	Not applicable. See **EE.S-IC.1–2**.
S-CP.8. (+) Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B\|A) = P(B)P(A\|B)$, and interpret the answer in terms of the model.	Not applicable.

CCSS Grade-Level Standards	DLM Essential Elements
S-CP.9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.	Not applicable.

High School Mathematics Domain: Statistics and Probability—Using Probability to Make Decisions

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Calculate expected values, and use them to solve problems.	
S-MD.1. (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.	Not applicable.
S-MD.2. (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.	Not applicable.
S-MD.3. (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*	Not applicable.
S-MD.4. (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*	Not applicable.
CLUSTER: Use probability to evaluate outcomes of decisions.	
S-MD.5. (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.	Not applicable.
S-MD.5.a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant.*	
S-MD.5.b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*	
S-MD.6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).	Not applicable.
S-MD.7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	Not applicable.