



DYNAMIC LEARNING MAPS ESSENTIAL ELEMENTS

FOR

Mathematics

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Dynamic Learning Maps Consortium (2013). *Dynamic Learning Maps Essential Elements for English Language Arts*. Lawrence, KS: University of Kansas.

and

Dynamic Learning Maps Consortium (2013). *Dynamic Learning Maps Essential Elements for Mathematics*. Lawrence, KS: University of Kansas.

Background on the Dynamic Learning Maps Essential Elements

The Dynamic Learning Maps Essential Elements are specific statements of knowledge and skills linked to the grade-level expectations identified in the Common Core State Standards. The purpose of the Dynamic Learning Maps Essential Elements is to build a bridge from the content in the Common Core State Standards to academic expectations for students with the most significant cognitive disabilities. The initial draft of the Dynamic Learning Maps Essential Elements (then called the Common Core Essential Elements) was released in the spring of 2012.

The initial version of the Dynamic Learning Maps Essential Elements (DLM EEs) was developed by a group of educators and content specialists from the 12 member states of the Dynamic Learning Maps Alternate Assessment Consortium (DLM) in the spring of 2011. Led by Edvantia, Inc., a sub-contractor of DLM, representatives from each state education agency and the educators and content specialists they selected developed the original draft of DLM EEs. Experts in mathematics and English language arts, as well as individuals with expertise in instruction for students with significant cognitive disabilities reviewed the draft documents. Edvantia then compiled the information into the version released in the spring of 2012.

Concurrent with the development of the DLM EEs, the DLM consortium was actively engaged in building learning maps in mathematics and English language arts. The DLM learning maps are highly connected representations of how academic skills are acquired, as reflected in research literature. In the case of the DLM project, the Common Core State Standards helped to specify academic targets, while the surrounding map content clarified how students could reach the specified standard. Learning maps of this size had not been previously developed, and as a

result, alignment between the DLM EEs and the learning maps was not possible until the fall of 2012, when an initial draft of the learning maps was available for review.

Alignment of the DLM EEs to the DLM Learning Maps

Teams of content experts worked together to revise the initial version of the DLM EEs and the learning maps to ensure appropriate alignment of these two critical elements of the project. Alignment involved horizontal alignment of the DLM EEs with the Common Core State Standards and vertical alignment of the DLM EEs with meaningful progressions in the learning maps. The alignment process began when researchers Caroline Mark and Kelli Thomas compared the learning maps with the initial version of the DLM EEs to determine how the map and the DLM EEs should be adjusted to improve their alignment. The teams of content experts most closely involved with this alignment work included:

Mathematics

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English Language Arts

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Suzanne Peterson

These teams worked in consultation with Sue Bechard, Ph.D. and Karen Erickson, Ph.D., who offered guidance based on their experience in alternate assessments of students with significant cognitive disabilities.

The Alignment Process

The process of aligning the learning map and the DLM EEs began by identifying nodes in the maps that represented the essential elements in mathematics and English language arts. This process revealed areas in the maps where additional nodes were needed to account for incremental growth reflected from an essential element in one grade to the next. Also identified

were areas in which an essential element was out of place developmentally, according to research, with other essential elements. For example, adjustments were made when an essential element related to a higher-grade map node appeared earlier on the map than an essential element related to a map node from a lower grade (e.g., a fifth-grade skill preceded a third-grade skill). Finally, the alignment process revealed DLM EEs that were actually written as instructional tasks rather than learning outcomes.

This initial review step provided the roadmap for subsequent revision of both the learning maps and the DLM EEs. The next step in the DLM project was to develop the claims document, which served as the basis for the evidence-centered design of the DLM project and helped to further refine both the modeling of academic learning in the maps and the final revisions to the DLM EEs.

Claims and Conceptual Areas

The DLM system uses a variant of evidence-centered design (ECD) as the framework for developing the DLM Alternate Assessment System. While ECD is multifaceted, it starts with a set of claims regarding important knowledge in the domains of interest (mathematics and English language arts), as well as an understanding of how that knowledge is acquired. Two sets of claims have been developed for DLM that identify the major domains of interest within mathematics and English language arts for students with significant cognitive disabilities. These claims are broad statements about expected student learning that serve to focus the scope of the assessment. Because the learning map identifies particular paths to the acquisition of academic skills, the claims also help to organize the structures in the learning map for this population of students. Specifically, conceptual areas within the map further define the knowledge and skills required to meet the broad claims identified by DLM.

The claims are also significant because they provide another means through which to evaluate alignment between the DLM EEs and the learning map nodes, and serve as the foundation for evaluating the validity of inferences made from test scores. DLM EEs related to a particular claim and conceptual area must clearly link to one another, and the learning map must reflect how that knowledge is acquired. Developing the claims and conceptual areas for DLM provided a critical framework for organizing nodes on the learning maps and, accordingly, the DLM EEs that align with each node.

The table below reveals the relationships among the claims, conceptual areas, and DLM EEs in mathematics. The DLM EEs are represented with codes that reflect the domains in mathematics. For example, the first letter or digit represents the grade of record, the next code reflects the domain, followed by the number that aligns with the Common Core State Standard grade level expectation. As such, K.CC.1 is the code for the DLM EE that aligns with kindergarten (K), counting and cardinality (CC), standard 1. Keys to the codes can be found under the table.

Clearly articulated claims and conceptual areas for DLM served as an important evidence-centered framework within which this version of the DLM EEs was developed. With the claims and conceptual areas in place, the relationship between DLM EEs within a claim and conceptual area or across grade levels is easier to track and strengthen. The learning maps, as well as the claims and conceptual areas, had not yet been developed when the original versions of the DLM EEs were created. As such, the relationship of DLM EEs within and across grade levels was more difficult to evaluate at that time.

Table 1. Dynamic Learning Maps Claims and Conceptual Areas for Students with Significant Cognitive Disabilities in Mathematics

<p>Claim 1</p>	<p>Number Sense: Students demonstrate increasingly complex understanding of number sense.</p> <p>Conceptual Areas in the Dynamic Learning Map:</p> <p>MC 1.1 Understand number structures (counting, place value, fraction) <i>Essential Elements Included:</i> K.CC.1, 4, 5; 1.NBT.1a-b; 2.NBT.2a-b,3; 3.NBT.1,2,3; 3.NF.1-3; 4.NF.1-2,3; 5.NF.1,2; 6.RP.1; 7.RP.1-3; 7.NS.2.c-d; 8.NS.2.a</p> <p>MC 1.2 Compare, compose, and decompose numbers and sets <i>Essential Elements Included:</i> K.CC.6; 1.NBT.2, 3, 4, 6; 2.NBT.1, 4, 5b; 4.NBT.2, 3; 5.NBT.1, 2, 3, 4; 6.NS.1, 5-8; 7.NS.3; 8.NS.2.b; 8.EE.3-4;</p> <p>MC 1.3 Calculate accurately and efficiently using simple arithmetic operations <i>Essential Elements Included:</i> 2.NBT.5.a, 6-7; 3.OA.4; 4.NBT.4, 5.NBT.5, 6-7; 6.NS.2, 3; 7.NS.1, 2.a, 2.b; 8.NS.1; 8.EE.1; N-CN.2.a, 2.b, 2.c; N-RN.1; S-CP.1-5; S-IC.1-2</p>
<p>Claim 2</p>	<p>Geometry: Students demonstrate increasingly complex spatial reasoning and understanding of geometric principles.</p> <p>Conceptual Areas in the Dynamic Learning Map:</p> <p>MC 2.1 Understand and use geometric properties of two- and three-dimensional shapes <i>Essential Elements Included:</i> K.MD.1-3; K.G.2-3; 1.G.1, 2; 2.G.1; 3.G.1; 4.G.1, 2; 4.MD.5, 6; 5.G.1-4; 5.MD.3; 7.G.1, 2, 3, 5; 8.G.1, 2, 4, 5; G-CO.1, 4-5, 6-8; G-GMD.1-3, 4</p> <p>MC 2.2 Solve problems involving area, perimeter, and volume <i>Essential Elements Included:</i> 1.G.3; 3.G.2; 4.G.3; 4.MD.3; 5.MD.4-5; 6.G.1, 2; 7.G.4, 6; 8.G.9; G-GMD.1-3; G-GPE.7</p>
<p>Claim 3</p>	<p>Measurement Data and Analysis: Students demonstrate increasingly complex understanding of measurement, data, and analytic procedures.</p> <p>Conceptual Areas in the Dynamic Learning Map:</p> <p>MC 3.1 Understand and use measurement principles and units of measure <i>Essential Elements Included:</i> 1.MD.1-2, 3.a, 3.b, 3.c, 3.d; 2.MD.1, 3-4, 5, 6, 7, 8; 3.MD.1, 2, 4; 4.MD.1, 2.a, 2.b, 2.c, 2.d; 5.MD.1.a, 1.b, 1.c; N-Q.1-3</p> <p>MC 3.2 Represent and interpret data displays <i>Essential Elements Included:</i> 1.MD.4; 2.MD.9-10; 3.MD.3; 4.MD.4.a, 4.b; 5.MD.2; 6.SP.1-2, 5; 7.SP.1-2, 3, 5-7; 8.SP.4; S-ID. 1-2, 3, 4</p>
<p>Claim 4</p>	<p>Algebraic and functional reasoning: Students solve increasingly complex mathematical problems, making productive use of algebra and functions.</p> <p>Conceptual Areas in the Dynamic Learning Map:</p> <p>MC 4.1. Use operations and models to solve problems <i>Essential Elements Included:</i> K.OA.1, 1.a, 1.b, 2, 5.a, 5.b; 2.OA.3, 4; 3.OA.1-2, 8; 4.OA.1-</p>

	<p>2, 3, 4; 6.EE.1-2, 3, 5-7; 7.EE.1, 4; 8.EE.7; A-CED.1, 2-4; A-SSE.1, 3</p> <p>MC 4.2 Understand patterns and functional thinking</p> <p><i>Essential Elements Included: 3.OA.9; 4.OA.5; 5.OA.3; 7.EE.2; 8.EE.5-6; 8.F.1-3, 4, 5; A-REI.10-12; A-SSE.4; F-BF.1, 2; F-IF.1-3, 4-6; F-LE.1</i></p>
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A-CED = creating equations; A-SSE = seeing structure in equations BF = building functions; CC = counting & cardinality; EE = expressions & equations; F-BF = basic fractions; F-IF = interpreting functions; G = geometry; G-GMD = geometric measurement & dimension; G-GPE = general properties & equations; MD = measurement & data; NBT = numbers & operations in base ten; N-CN = complex number system; NF = numbers & operations - fractions; N-RN = real number system; NS = number systems; N-Q = number & quantity; OA = operations & algebraic thinking; RP = ratios & proportional relationships; S-IC- statistics & probability - making inferences/justifying conclusions; S-ID = statistics & probability - interpreting categorical & quantitative data; SP = statistics & probability

Resulting Changes to the DLM Essential Elements

The development of the entire DLM Alternate Assessment System guided a final round of revisions to the DLM EEs, which can be organized into four broad categories: alignment across grade levels, language specificity, common core alignment, and defining learning expectations (rather than instructional tasks). The first type of revision was required to align the DLM EEs across grade levels, both vertically and horizontally. The maps, and the research supporting them, were critical in determining the appropriate progression of skills and understandings from grade to grade. This alignment across grade levels was important within and across standards, strands, and domains. For example, in determining when it was appropriate to introduce concepts in mathematics regarding the relative position of objects, we had to consider the grade level at which prepositions that describe relative position were introduced in English language arts. Examining the research-based skill development outlined in the learning map aided in these kinds of determinations.

The articulation of the claims and conceptual areas reinforced the need for specific language in the DLM EEs to describe learning within an area. Because teams assigned to grade bands developed the first round of DLM EEs, the language choices from one grade to the next were not consistent. Even when closely related skills, concepts, or understandings were

targeted, the same terms were not always selected to describe the intended learning outcome. The teams of content experts who worked on this revised version of the DLM EEs were very intentional in selecting a common set of terms to reflect the claims and conceptual areas and applied them consistently across the entire set of DLM EEs.

Another important change in this version of the DLM EEs involved alignment to the Common Core State Standards (CCSS). Given that the DLM EEs are intended to clarify the bridge to the CCSS expectations for students with the most significant cognitive disabilities, it is critical that alignment be as close as possible without compromising learning and development over time. While there was never a one-to-one correspondence between the CCSS and the DLM EEs, the revisions have made the alignment between the two more precise than it was in the first version.

Finally, revisions to the DLM EEs involved shifting the focus of a small number of DLM EEs that were written in the form of instructional tasks rather than learning expectations, and adding “With guidance and support” to the beginning of a few of the DLM EEs in the primary grades in English language arts to reflect the expectations articulated in the CCSS.

Members of the DLM consortium reviewed each of the changes to the original version of the DLM EEs. Four states provided substantive feedback on the revisions, and this document incorporates the changes those teams suggested.

Access to Instruction and Assessment

The DLM EEs specify learning targets for students with significant cognitive disabilities; however, they do not describe all of the ways that students can engage in instruction or demonstrate understanding through an assessment. Appropriate modes of communication, both

for presentation or response, are not stated in the DLM EEs unless a specific mode is an expectation. Where no limitation has been stated, no limitation should be inferred. Students' opportunities to learn and to demonstrate learning during assessment should be maximized by providing whatever communication, assistive technologies, augmentative and alternative communication (AAC) devices, or other access tools that are necessary and routinely used by the student during instruction.

Students with significant cognitive disabilities include a broad range of students with diverse disabilities and communication needs. For some students with significant cognitive disabilities, a range of assistive technologies is required to access content and demonstrate achievement. For other students, AAC devices or accommodations for hearing and visual impairments will be needed. During instruction, teams should meet individual student needs using whatever technologies and accommodations are required. Examples of some of the ways that students may use technology while learning and demonstrating learning are topics for professional development, and include:

- communication devices that compensate for a student's physical inability to produce independent speech.
- alternate access devices that compensate for a student's physical inability to point to responses, turn pages in a book, or use a pencil or keyboard to answer questions or produce writing.

Guidance and Support

The authors of the CCSS use the words "prompting and support" at the earliest grade levels to indicate when students are not expected to achieve standards completely independently. Generally, "prompting" refers to "the action of saying something to persuade,

encourage, or remind someone to do or say something” (McKean, 2005). However, in special education, prompting is often used to mean a system of structured cues to elicit desired behaviors that otherwise would not occur. In order to clearly communicate that teacher assistance is permitted during instruction of the DLM EEs and is not limited to structured prompting procedures, the decision was made by the stakeholder group to use the more general term *guidance* throughout the DLM EEs.

Guidance and support during instruction should be interpreted as teacher encouragement, general assistance, and informative feedback to support the student in learning. Some examples of the kinds of teacher behaviors that would be considered guidance and support include verbal supports, such as

- getting the student started (e.g., “Tell me what to do first.”),
- providing a hint in the right direction without revealing the answer (e.g., Student wants to write *dog* but is unsure how, so the teacher might say, “See if you can write the first letter in the word, /d/og [phonetically pronounced].”),
- using structured technologies such as task-specific word banks, or
- providing structured cues such as those found in prompting procedures (e.g., least-to-most prompts, simultaneous prompting, and graduated guidance).

Guidance and support as described above applies to instruction and is also linked to demonstrating learning relative to DLM EEs, where guidance and support is specifically called out within the standards.

Conclusion

Developing the research-based model of knowledge and skill development represented in the DLM Learning Maps supported the articulation of assessment claims for mathematics and English language arts. This articulation subsequently allowed for a careful revision of the DLM EEs to reflect both horizontal alignment with the CCSS and vertical alignment across the grades, with the goal of moving students toward more sophisticated understandings in both domains. Though the contributions made by Edvantia and our state partners in developing the initial set of DLM EEs were a critical first step, additional revisions to the DLM EEs were required to ensure consistency across all elements of the Dynamic Learning Maps Alternate Assessment System.

APPENDIX

Development of the Dynamic Learning Maps Essential Elements has been a collaborative effort among practitioners, researchers, and our state representatives. Listed below are the reviews and the individuals involved with each round of improvements to the Dynamic Learning Maps Essential Elements. Thank you to all of our contributors.

Review of Draft Two of Dynamic Learning Maps Essential Elements

A special thanks to all of the experts nominated by their state to review draft two of the Dynamic Learning Maps Essential Elements. We are grateful for your time and efforts to improve these standards for students with significant cognitive disabilities. Your comments have been incorporated into this draft. The states with teams who reviewed draft two include:

Illinois	Oklahoma
Iowa	Utah
Kansas	Virginia
Michigan	West Virginia
Missouri	Wisconsin

Development of the Original Dynamic Learning Maps Common Core Essential Elements

A special thanks to Edvantia and the team of representatives from Dynamic Learning Maps consortium states who developed the original Common Core Essential Elements upon which the revised Dynamic Learning Maps Essential Elements are based. The team from Edvantia who led the original effort included:

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Nathan Davis, Information Technology Specialist
Kristen Deitrick, Corporate Communications Specialist

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Representatives from Dynamic Learning Maps consortium states included:

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SEA Representatives: Tom Deeter, Emily Thatcher

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DYNAMIC LEARNING MAPS ESSENTIAL ELEMENTS FOR SIXTH GRADE

Sixth Grade Mathematics Domain: Ratios and Proportional Relationships

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Understand ratio concepts, and use ratio reasoning to solve problems.	
6.RP.1. Understand the concept of a ratio, and use ratio language to describe a ratio relationship between two quantities. <i>For example, “The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”</i>	EE.6.RP.1. Demonstrate a simple ratio relationship.
6.RP.2. Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship. <i>For example, “This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is $3/4$ cup of flour for each cup of sugar.” “We paid \$75 for 15 hamburgers, which is a rate of \$5 per hamburger.”¹⁹</i>	Not applicable. See EE.7.RP.1–3.
6.RP.3. Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.	Not applicable. See EE.8.F.1–3.
6.RP.3.a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.	
6.RP.3.b. Solve unit rate problems including those involving unit pricing and constant speed. <i>For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?</i>	
6.RP.3.c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means $30/100$ times the quantity); solve problems involving finding the whole, given a part and the percent.	
6.RP.3.d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	

¹⁹ Expectations for unit rates in this grade are limited to non-complex fractions.

Sixth Grade Mathematics Domain: The Number System

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	
<p>6.NS.1. Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. <i>For example, create a story context for $(2/3) \div (3/4)$, and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that $(2/3) \div (3/4) = 8/9$ because $3/4$ of $8/9$ is $2/3$. (In general, $(a/b) \div (c/d) = ad/bc$.) How much chocolate will each person get if 3 people share $1/2$ lb. of chocolate equally? How many $3/4$-cup servings are in $2/3$ of a cup of yogurt? How wide is a rectangular strip of land with length $3/4$ mi and area $1/2$ square mi?</i></p>	<p>EE.6.NS.1. Compare the relationships between two unit fractions.</p>
CLUSTER: Compute fluently with multi-digit numbers, and find common factors and multiples.	
<p>6.NS.2. Fluently divide multi-digit numbers using the standard algorithm.</p>	<p>EE.6.NS.2. Apply the concept of fair share and equal shares to divide.</p>
<p>6.NS.3. Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.</p>	<p>EE.6.NS.3. Solve two-factor multiplication problems with products up to 50 using concrete objects and/or a calculator.</p>
<p>6.NS.4. Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. <i>For example, express $36 + 8$ as $4(9 + 2)$.</i></p>	<p>Not applicable.</p>

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Apply and extend previous understandings of numbers to the system of rational numbers.	
<p>6.NS.5. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.</p>	<p>EE.6.NS.5–8. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero).</p>
<p>6.NS.6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.</p>	
<p>6.NS.6.a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., $-(-3) = 3$, and that 0 is its own opposite.</p>	
<p>6.NS.6.b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.</p>	
<p>6.NS.6.c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.</p>	
<p>6.NS.7. Understand ordering and absolute value of rational numbers.</p>	
<p>6.NS.7.a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. <i>For example, interpret $-3 > -7$ as a statement that -3 is located to the right of -7 on a number line oriented from left to right.</i></p>	
<p>6.NS.7.b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. <i>For example, write $-3^{\circ}C > -7^{\circ}C$ to express the fact that $-3^{\circ}C$ is warmer than $-7^{\circ}C$.</i></p>	
<p>6.NS.7.c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. <i>For example, for an account balance of -30 dollars, write $-30 = 30$ to describe the size of the debt in dollars.</i></p>	
<p>6.NS.7.d. Distinguish comparisons of absolute value from statements about order. <i>For example, recognize that an account balance less than -30 dollars represents a debt greater than 30 dollars.</i></p>	

CCSS Grade-Level Standards	DLM Essential Elements
6.NS.8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	EE.6.NS.5–8. Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero).

Sixth Grade Mathematics Domain: Expressions and Equations

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Apply and extend previous understandings of arithmetic to algebraic expressions.	
6.EE.1. Write and evaluate numerical expressions involving whole-number exponents.	EE.6.EE.1–2. Identify equivalent number sentences.
6.EE.2. Write, read, and evaluate expressions in which letters stand for numbers.	
6.EE.2.a. Write expressions that record operations with numbers and with letters standing for numbers. <i>For example, express the calculation “Subtract y from 5” as $5 - y$.</i>	
6.EE.2.b. Identify parts of an expression using mathematical terms (<i>sum, term, product, factor, quotient, coefficient</i>); view one or more parts of an expression as a single entity. <i>For example, describe the expression $2(8 + 7)$ as a product of two factors; view $(8 + 7)$ as both a single entity and a sum of two terms.</i>	
6.EE.2.c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). <i>For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.</i>	
6.EE.3. Apply the properties of operations to generate equivalent expressions. <i>For example, apply the distributive property to the expression $3(2 + x)$ to produce the equivalent expression $6 + 3x$; apply the distributive property to the expression $24x + 18y$ to produce the equivalent expression $6(4x + 3y)$; apply properties of operations to $y + y + y$ to produce the equivalent expression $3y$.</i>	EE.6.EE.3. Apply the properties of addition to identify equivalent numerical expressions.
6.EE.4. Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). <i>For example, the expressions $y + y + y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.</i>	Not applicable.

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Reason about and solve one-variable equations and inequalities.	
<p>6.EE.5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.</p>	<p>EE.6.EE.5–7. Match an equation to a real-world problem in which variables are used to represent numbers.</p>
<p>6.EE.6. Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.</p>	
<p>6.EE.7. Solve real-world and mathematical problems by writing and solving equations of the form $x + p = q$ and $px = q$ for cases in which p, q and x are all nonnegative rational numbers.</p>	
<p>6.EE.8. Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x < c$ have infinitely many solutions; represent solutions of such inequalities on number line diagrams.</p>	<p>Not applicable.</p>
CLUSTER: Represent and analyze quantitative relationships between dependent and independent variables.	
<p>6.EE.9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. <i>For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.</i></p>	<p>Not applicable.</p>

Sixth Grade Mathematics Domain: Geometry

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Solve real-world and mathematical problems involving area, surface area, and volume.	
<p>6.G.1. Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.</p>	<p>EE.6.G.1. Solve real-world and mathematical problems about area using unit squares.</p>
<p>6.G.2. Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.</p>	<p>EE.6.G.2. Solve real-world and mathematical problems about volume using unit cubes.</p>
<p>6.G.3. Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.</p>	<p>Not applicable.</p>
<p>6.G.4. Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.</p>	<p>Not applicable.</p>

Sixth Grade Mathematics Domain: Statistics and Probability

CCSS Grade-Level Standards	DLM Essential Elements
CLUSTER: Develop understanding of statistical variability.	
<p>6.SP.1. Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. <i>For example, “How old am I?” is not a statistical question, but “How old are the students in my school?” is a statistical question because one anticipates variability in students’ ages.</i></p>	<p>EE.6.SP.1–2. Display data on a graph or table that shows variability in the data.</p>
<p>6.SP.2. Understand that a set of data collected to answer a statistical question has a distribution, which can be described by its center, spread, and overall shape.</p>	
<p>6.SP.3. Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.</p>	<p>Not applicable. See EE.S-ID.4.</p>
CLUSTER: Summarize and describe distributions.	
<p>6.SP.4. Display numerical data in plots on a number line, including dot plots, histograms, and box plots.</p>	<p>Not applicable. See EE.6.SP.1–2.</p>
<p>6.SP.5. Summarize numerical data sets in relation to their context, such as by:</p>	<p>EE.6.SP.5. Summarize data distributions shown in graphs or tables.</p>
<p>6.SP.5.a. Reporting the number of observations.</p>	
<p>6.SP.5.b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.</p>	
<p>6.SP.5.c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.</p>	
<p>6.SP.5.d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.</p>	